

# Digital Image Processing and Pattern Recognition

E1528

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Lecture 5



## Fundamentals of Spatial Filtering (Correlation and Convolution)

**INSTRUCTOR**

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## ➤ Introduction

- Spatial filtering is used in a **broad spectrum** of image processing applications, so a solid understanding of filtering principles is important.
- The name **filter** is borrowed from frequency domain processing where “**filtering**” refers to **passing**, **modifying**, or **rejecting** specified frequency components of an image.
- For example, a filter that passes **low frequencies** is called a **lowpass filter**. The net effect produced by a lowpass filter is to smooth an image by blurring it. We can accomplish similar smoothing directly on the image itself by using **spatial filters**.

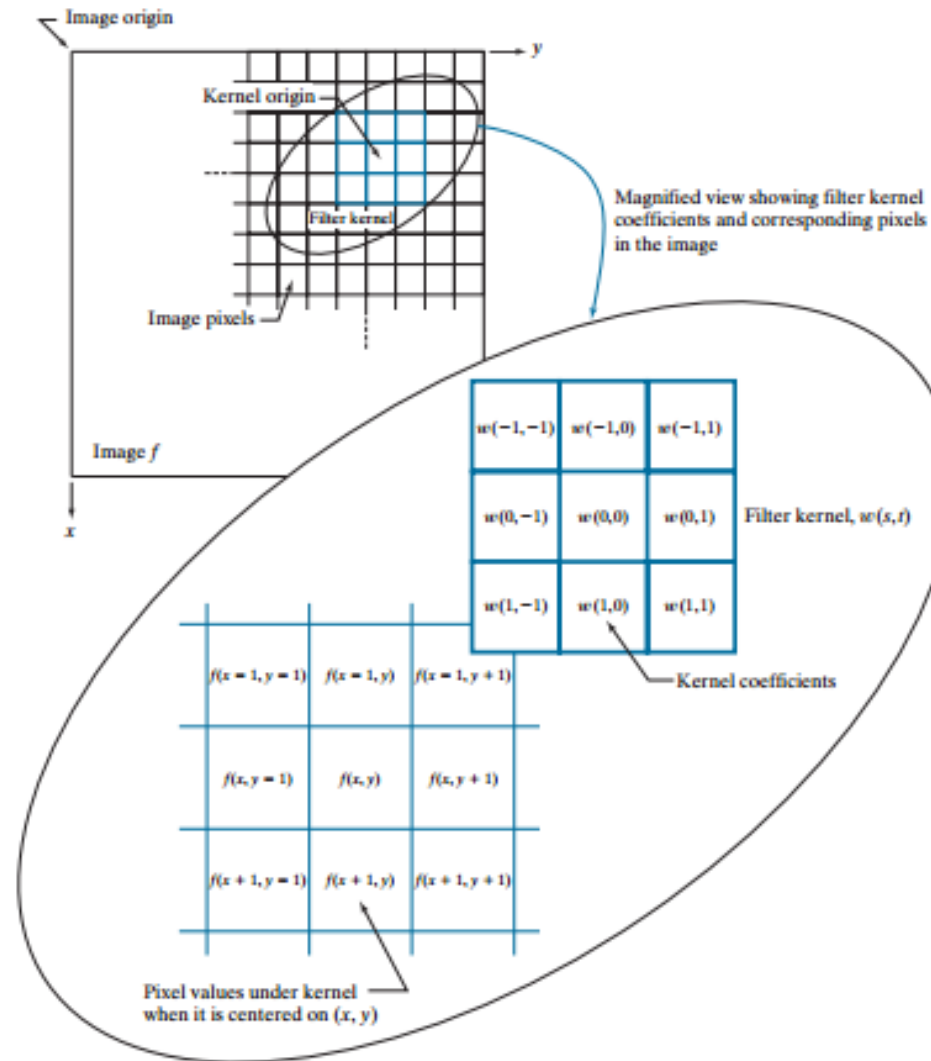
## ➤ Spatial Filters

- Spatial filtering modifies an image by **replacing the value of each pixel by a function** of the values of the pixel and its neighbors.
- If the operation performed on the image pixels is **linear**, then the filter is called a **linear spatial filter**. Otherwise, the filter is a **nonlinear** spatial filter.
- We will focus attention first on linear filters and then introduce some basic nonlinear filters.

## ➤ The Mechanics of Linear Spatial Filtering

- A linear spatial filter performs a **sum-of-products** operation between an image  $f$  and a filter kernel,  $w$ .
- The **kernel** is an array whose size defines the neighborhood of operation, and whose coefficients determine the nature of the filter.
- Other terms used to refer to a spatial filter **kernel** are **mask**, **template**, and **window**. We use the term **filter kernel** or simply **kernel**.

# ➤ The Mechanics of Linear Spatial Filtering



## ➤ The Mechanics of Linear Spatial Filtering

- Last figure illustrates the mechanics of linear spatial filtering using a  $3 \times 3$  kernel. At any point  $(x,y)$  in the image, the response,  $g(x,y)$ , of the filter is the sum of products of the kernel coefficients and the image pixels encompassed by the kernel:

$$\begin{aligned} g(x,y) &= w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots + w(0,0)f(x,y) + \dots \\ &+ w(1,1)f(x+1,y+1) \end{aligned}$$

- As coordinates  $x$  and  $y$  are varied, the center of the kernel moves from pixel to pixel, generating the filtered image,  $g$ , in the process.

## ➤ The Mechanics of Linear Spatial Filtering

- Observe that the center coefficient of the kernel,  $w(0,0)$ , aligns with the pixel at location  $(x,y)$ . For a kernel of size  $m \times n$ , we assume that  $m = 2a+1$  and  $n = 2b+1$ , where  $a$  and  $b$  are nonnegative integers.
- This means that our focus is on kernels of **odd** size in both coordinate directions.
- In general, linear spatial filtering of an image of size  $M \times N$  with a kernel of size  $m \times n$  is given by the expression

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t) \quad (1)$$



## ➤ Spatial Correlation and Convolution

- **Spatial correlation** is illustrated graphically in last Figure, and it is described mathematically by equation (1).
- Correlation consists of **moving the center** of a kernel over an image and computing the **sum of products** at each location.
- The mechanics of **spatial convolution** are the same, except that the **correlation** kernel is **rotated by 180°**. Thus, when the values of a kernel are **symmetric about its center**, correlation and convolution yield the **same** result. The reason for rotating the kernel will become clear in the following discussion. The best way to explain the differences between the two concepts is by example.

## ➤ Spatial Correlation and Convolution (cont.)

- We begin with a **1-D** illustration, in which case equation(1) becomes

$$g(x) = \sum_{s=-a}^a w(s)f(x + s) \quad (2)$$

Next figure (a) shows a 1-D function,  $f$ , and a kernel,  $w$ . The kernel is of size  $1 \times 5$ , so  $a = 2$  and  $b = 0$  in this case. Figure (b) shows the starting position used to perform correlation, in which  $w$  is positioned so that its center coefficient is coincident with the origin of  $f$ .

## ➤ Spatial Correlation and Convolution (cont.)

- The first thing we notice is that part of  $w$  lies outside  $f$ , so the summation is undefined in that area.
- A solution to this problem is to **pad function**  $f$  with enough 0's on either side. In general, if the kernel is of size  $1 \times m$ , we need  $(m-1)/2$  zeros on either side of  $f$  in order to handle the beginning and ending configurations of  $w$  with respect to  $f$ .
- Figure(c) shows a properly padded function. In this starting configuration, all coefficients of the kernel overlap valid values.

# ➤ Spatial Correlation and Convolution (cont.)

## Correlation

(a)  $\begin{matrix} \swarrow & \text{Origin} & & f & & & & & & w \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 4 & 2 & 8 \end{matrix}$

(b)  $\begin{matrix} & & & \downarrow & & & & & & & & & & & \\ & & & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & & \\ 1 & 2 & 4 & 2 & 8 & & & & & & & & & & \\ & & & \uparrow & & & & & & & & & & & \\ & & & \text{Starting position alignment} & & & & & & & & & & & \end{matrix}$

(c)  $\begin{matrix} & & & \text{Zero padding} & & & & & & & & & & & \\ & \swarrow & & & & \searrow & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 1 & 2 & 4 & 2 & 8 & & & & & & & & & & \\ & & & \uparrow & & & & & & & & & & & \\ & & & \text{Starting position} & & & & & & & & & & & \end{matrix}$

## Convolution

(i)  $\begin{matrix} \swarrow & \text{Origin} & & f & & & & & & w \text{ rotated } 180^\circ \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 8 & 2 & 4 & 2 & 1 \end{matrix}$  (i)

(j)  $\begin{matrix} & & & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & & \\ 8 & 2 & 4 & 2 & 1 & & & & & & & & & & \\ & & & \uparrow & & & & & & & & & & & \\ & & & \text{Starting position alignment} & & & & & & & & & & & \end{matrix}$  (j)

(k)  $\begin{matrix} & & & \text{Zero padding} & & & & & & & & & & & \\ & \swarrow & & & & \searrow & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 8 & 2 & 4 & 2 & 1 & & & & & & & & & & \\ & & & \uparrow & & & & & & & & & & & \\ & & & \text{Starting position} & & & & & & & & & & & \end{matrix}$  (k)

## ➤ Spatial Correlation and Convolution (cont.)

(d)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $1\ 2\ 4\ 2\ 8$   
 ↑ Position after 1 shift

(l)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $8\ 2\ 4\ 2\ 1$   
 ↑ Position after 1 shift

(e)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $1\ 2\ 4\ 2\ 8$   
 ↑ Position after 3 shifts

(m)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $8\ 2\ 4\ 2\ 1$   
 ↑ Position after 3 shifts

(f)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $1\ 2\ 4\ 2\ 8$   
 Final position ↑

(n)  $0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0$   
 $8\ 2\ 4\ 2\ 1$   
 Final position ↑

**Correlation result**

**Convolution result**

(g)  $0\ 8\ 2\ 4\ 2\ 1\ 0\ 0$

(o)  $0\ 1\ 2\ 4\ 2\ 8\ 0\ 0$

**Extended (full) correlation result**

**Extended (full) convolution result**

(h)  $0\ 0\ 0\ 8\ 2\ 4\ 2\ 1\ 0\ 0\ 0\ 0$

(p)  $0\ 0\ 0\ 1\ 2\ 4\ 2\ 8\ 0\ 0\ 0\ 0$



## ➤ Properties

- Some fundamental properties of convolution and correlation.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

- Sometimes an image is filtered (i.e., convolved) sequentially, in stages say Q stage. Because of the **commutative** property of convolution, this multistage filtering can be done in a single filtering operation, as following.
- Note that We cannot do the same reduction for correlation because it is not commutative.

## ➤ **Padding**

- What is Padding?
- Types of padding.
- Comparison



## ➤ What is Padding?

- **Padding** extends the boundaries of an image **to avoid undefined operations** when parts of a kernel (mask) **lie outside** the border of the image during filtering.
- In general, if the kernel is of size **m\*n**, we need **(m-1)/2** rows at the top and bottom and **(n-1)/2** columns at the right and left and the values these new cells will take depends on the padding technique.

## ➤ **Types of padding**

- Zero padding
- Mirror (or symmetric) padding
- Replicate padding

## ➤ zero padding

- In signal processing, zero padding refers to the practice of **adding zeroes** to a time-domain signal.
- Zero-padding is often done before performing a fast Fourier transform on the time-domain signal.

$$\mathbf{X} = \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix},$$

$$\text{Pad}(1, \mathbf{X}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & a & b & c & 0 \\ 0 & d & f & g & 0 \\ 0 & h & j & k & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

## ➤ Mirror padding

- Values outside the boundary of the image are obtained by mirror-reflecting the image across its border.

13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8

## ➤ Replicate padding

- Values outside the boundary are set equal to the nearest image border value. It is useful when the areas near the border of the image are constant.

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

## ➤ Comparison

- **Zero Padding:** When zero (**black**) padding is used, the net result of smoothing at or **near** the border is a **dark** gray border that arises from including black pixels in the averaging process.
- Using the 11\*11 kernel resulted in a more prominent dark border. The result with the 21\*21 the kernel shows significant blurring of all components of the image, including a darker boarder.
- The **two** other approaches to padding tend to **solve the dark-border** problem that results from zero padding.

## ➤ Comparison

- **Mirror Padding:** mirror padding does duplicate the details of the image on the edges which makes it more applicable when the areas near the border contain image details.



- **Replicate Padding:** It duplicate the only pixel on the edge which makes it useful when the areas near the border of the image are constant.

Thank  
you

